Chapter 9

Propagation Channels

The transmit and receive antennas in the systems we have analyzed in earlier chapters have been in free space with no other objects present. In a practical communication system, the quality of the channel can be influenced significantly by materials in the environment that cause blockage, scattering and multipath. Common scattering and multipath effects include:

> *Reflection from earth:* Soil and rock are lossy dielectrics and reflect RF and microwave signals. A simple example of a non-free space propagation environment is a direct path from transmitter to receiver and a reflection from the ground.
>
> *Slow fading:* Blockage or shadowing by mountains, buildings, structures, or other objects between the transmitter and receiver, causing signal amplitude variation with a long spatial scale as the transmitter or the receiver moves.
>
> *Diffraction:* Mountain peaks, building corners, or other structures with sharp edges can lead to diffraction, so that a signal spreads into the shadow of the object, and the shadow boundary is gradual rather than abrupt.
>
> *Fast fading:* Interference due to multipath cases cancelation and nulls on a short spatial scale on the order of a wavelength in size, due to changes in the relative phases of the multiple reflected signals as the transmitter, receiver, or scatterers move. In addition to nulls in intensity, multiple time-delayed copies of the transmitted signal are received, leading to channel quality degradation.
>
> *Doppler shift.* Moving environments or moving transceivers introduce Doppler shift, which means that the effect of the environment on modulated signals over a given operating bandwidth must be considered.

Simple propagation environments can be treated deterministically using closed form analysis. Complex scattering environments require more detailed numerical models, stochastic analyses, or Monte Carlo simulation. In this chapter, we will begin with simple models for multipath effects and move towards more complicated characterizations.

9.1 Path Loss

A simple, empirical model for a propagation environment can be obtained by characterize the effect of blockage, fading, and other effects in terms of long term averaged signal reduction versus distance between the transmitter and receiver. In free space, signal power decreases as $1/r^2$ with distance from the transmitter.

In a complex environment, signal strength can vary rapidly with position as the receiver moves through nulls, fast fades, and diffraction regions. An environment with few scatterers will behave similarly to free space on average, whereas many scatterers lead to a greater likelihood of fading and hence more rapid decay of signal strength with distance.

As a very high level, rough model for propagation effects, detailed, local variability in signal strength can be ignored or averaged out, and a simple form for the average signal strength extracted from measured data. This approximate behavior can be parameterized by modifying the Friis transmission formula to have an exponent on the distance between receiver and transmitter that varies depending on the environment, so that

$$P_{\rm rec} = P_{\rm t} G_{\rm r} G_{\rm t} \frac{a}{r^n} \tag{9.1}$$

where a is a constant. Typical values for the path loss exponent n are shown below.

Environment	Path loss exponent n
Free space	2
Flat rural	3
Rolling rural	3.5
Suburban	4
Dense urban	4.5

In some situations, high path loss can be beneficial, to allow frequency reuse from one city or region to another. For AM and FM radio, for example, frequencies are reused in different cities based on rules that avoid too much interference between stations. In other cases, high path loss means that repeaters, satellite links, or other means must be used to overcome loss due to blockage and scattering between the transmitter and receiver.

9.2 Plane Earth Model

One of the most basic of all propagation models is a transmitter and receiver each at some distance above ground and a give separation. We will approximate the ground as flat to simplify the model. A deterministic multipath model for a transmitter and receiver above ground can be obtained by adding fields due to a direct propagation path and reflection from flat earth.

Consider an antenna at height h_t and receiver at height h_r above a flat earth, with a horizontal separation R. The electric field at the receiver is the sum of the line of sight ray and the reflection. The phases of the two paths are determined by the propagation distances,

$$\begin{aligned} r_d &= \sqrt{R^2 + (h_t - h_r)^2} \\ r_e &= \sqrt{R^2 + (h_t + h_r)^2} \end{aligned}$$

The path difference can be approximated by

$$\begin{aligned} \Delta r &= \sqrt{R^2 + (h_t + h_r)^2} - \sqrt{R^2 + (h_t - h_r)^2} \\ &\simeq R\left(1 + \frac{(h_t + h_r)^2}{2R^2}\right) - R\left(1 + \frac{(h_t - h_r)^2}{2R^2}\right) \\ &= \frac{(h_t + h_r)^2 - (h_t - h_r)^2}{2R} \\ &= \frac{2h_t h_r}{R} \end{aligned}$$

The path difference can be used to find the sum of the direct and reflected rays.

Ignoring the bulk phase of the signal over the common part of the path distance, the total electric field at the receiver is

$$\overline{E} = \overline{E}_d(1 + \Gamma e^{-jk\Delta r}) \tag{9.2}$$

where Γ is the reflection coefficient from the air-ground interface. We have assumed perpendicular polarization so that the direct and reflected electric fields are in the same direction. For near grazing incidence on an interface with a lossy dielectric, the reflection coefficient is $\Gamma \simeq -1$. The power density at the receiver is

$$\begin{aligned} S &= \frac{|\overline{E}|^2}{2\eta} \\ &= \frac{|\overline{E}_d|^2}{2\eta}|1 - e^{-jk\Delta r}|^2 \\ &= S_d 4\sin^2(k\Delta r/2) \\ &= S_d 4\sin^2(kh_t h_r/R) \end{aligned}$$

If the horizontal separation R is much greater than the antenna heights, then the power density simplifies to

$$S \simeq S_d \left(\frac{2kh_t h_r}{R}\right)^2$$

We can insert this result into the Friis transmission formula to obtain the received power,

$$\begin{aligned} P_{\text{rec}} &= S\frac{\lambda^2}{4\pi}G_r \\ &= S_d \left(\frac{2kh_t h_r}{R}\right)^2 \frac{\lambda^2}{4\pi}G_r \\ &= \frac{P_t}{4\pi r^2}G_t \left(\frac{2kh_t h_r}{R}\right)^2 \frac{\lambda^2}{4\pi}G_r \\ &= P_t G_t G_r \frac{h_t^2 h_r^2}{r^4} \end{aligned}$$

This expression provides a simple approximation to the field at a receiver when there is a direct and reflected path over flat earth.

This analysis shows that the path loss exponent for the plane earth model is $n = 4$. As the antenna heights become small, the received field goes to zero. The rapid decrease of field strength with separation distance and antenna height is due to the reflection coefficient $\Gamma \simeq -1$, which causes the direct and reflected paths to cancel as the path lengths become equal. In practice, the path loss exponent over flat earth is smaller, and the power density does not decay as rapidly as predicted by the plane earth model. If the surface is rough, then the reflected signal at the receiver is weaker and the cancelation between the direct and reflected signals is not as complete.

9.3 Classical Stochastic Multipath Models

The path loss exponent is only a very simple measure of the effect of materials and structures on a propagation channel. Complex propagation environments exhibit important characteristics on short time scales like fast fading that are not captured by this model. To treat multipath in a more sophisticated way, we need to model a propagation channel stochastically with random multipath due to moving objects or motion of the transmitter and receiver.

9.3.1 Rayleigh Channel Model

In a rich scattering environment such as an urban area, the signal reflects from buildings and other structures and travels from the transmitter to the receiver along many paths. This effect is referred to as multipath. The propagating waves along the various path travel different distances and scatter with different strengths at each obstacle. For this reason, the field arriving at the receiver location in a rich multipath environment can be characterized stochastically as the sum of waves with random amplitudes and phases. Furthermore, if the environment is highly cluttered, it is not likely that the signal will arrive predominantly from one direction, and there is no strong line of sight component in the received field. These considerations motivate the classical Rayleigh channel model.

At a point in space, the arriving field in general consists of multiple plane waves arriving in different directions, so that one component of the incident field intensity vector at the receive antenna can be represented as

$$E = \sum_{i=1}^{N} Z_i = X + jY \tag{9.3}$$

where Z_1, Z_2, …, Z_N are random variables representing the phase and magnitude of the electric field arriving from N different propagation paths. Each arriving field corresponds to a reflection or diffraction of the signal from an object in the propagation environment. The statistics of the random variables are determined by the properties of the propagation environment as it changes in time or as the transmitter and receiver move. The phases of the random variables Z_i represent the different lengths of the propagation paths. The reflections add coherently at the receiver, sometimes leading to a large magnitude for the total field E and at other times cancelation occurs and E is small. The goal of this analysis is to determine the statistics of $|E|$ in order to characterize the fast fading properties of the propagation channel.

A simple choice for the random variables Z_i are independent, zero mean, circular complex Gaussian random variables. This means that if we divide Z_i into real and imaginary parts, so that $Z_i = R_i + jS_i$, then R_i and S_i are independent, identically distributed (IID) Gaussian random variables. The choice of Gaussian random variables can be argued based on the central limit theorem, by comparison to measurements, or simply because it is mathematically convenient.

We first need to determine the statistics of the random variable E in terms of the statistics of Z_i. Clearly E is zero mean because it is the sum of zero mean random variables. The variances of the real and imaginary parts of E are

$$\mathrm{E}[X^2] = \mathrm{E}\left[\sum_i R_i \sum_k R_k\right] = \sum_{i,k} \mathrm{E}[R_i R_k] = \sum_i \mathrm{E}[R_i^2] = \sigma^2 \tag{9.4}$$

$$\mathrm{E}[Y^2] = \sigma^2 \tag{9.5}$$

The time-average power in the received signal is proportional to

$$P = \frac{\mathrm{E}[|E|^2]}{2} = \frac{\mathrm{E}[X^2] + \mathrm{E}[Y^2]}{2} = \sigma^2 \tag{9.6}$$

This can be thought of as the signal power at the receiver output relative to a $1\,\Omega$ load impedance and ignoring the scale factor that relates the electric field at the receiver and antenna output voltage. The correlation of the real and imaginary parts is

$$\mathrm{E}[XY] = \sum_{i,k} \mathrm{E}[R_i S_k] = 0 \tag{9.7}$$

since the real and imaginary parts of each of the random variables Z_i are independent.

Now let us look at the statistics of the magnitude and phase of the total electric field,

$$r = |E| = \sqrt{X^2 + Y^2} \tag{9.8}$$
$$\theta = \tan^{-1}(Y/X) \tag{9.9}$$

The joint probability distribution function (PDF) $f_{r\theta}(r, \theta)$ is related to the joint PDF $f_{XY}(X, Y)$ by

$$f_{r\theta}(r, \theta) = f_{XY}(X, Y) \underbrace{\det \begin{bmatrix} \frac{\partial X}{\partial r} & \frac{\partial Y}{\partial r} \\ \frac{\partial X}{\partial \theta} & \frac{\partial Y}{\partial \theta} \end{bmatrix}}_{\text{Jacobian}} \tag{9.10}$$

The Jacobian is

$$\det \begin{bmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{bmatrix} = r\cos^2\theta + r\sin^2\theta = r \tag{9.11}$$

Since X and Y are IID Gaussian random variables, the joint PDF is the product of the individual PDFs, which are both Gaussian, so that

$$f_{XY}(X, Y) = \frac{e^{-(X^2+Y^2)/(2\sigma^2)}}{2\pi\sigma^2} \tag{9.12}$$

Applying (9.10) leads to the joint distribution

$$f_{r\theta}(r, \theta) = \frac{r}{2\pi\sigma^2} e^{-r^2/(2\sigma^2)} \tag{9.13}$$

Since this expression is independent of θ, the phase is uniformly distributed on the interval $[0, 2\pi]$. If we integrate over θ, we obtain the PDF of the magnitude,

$$\begin{aligned} f_r(r) &= \int_0^{2\pi} \frac{r}{2\pi\sigma^2} e^{-r^2/(2\sigma^2)} d\theta \\ &= \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)} \end{aligned} \tag{9.14}$$

Since r is nonnegative, this expression is valid only for $r \geq 0$, and the PDF is zero for $r < 0$. This is in the form of a Rayleigh PDF. The statistical distribution of the magnitude of the field at the receiver in a rich multipath environment can therefore be modeled by the Rayleigh PDF.

The cumulative distribution function (CDF) of the magnitude is particularly useful, because we can use it to determine what fraction of the time the signal level is below the minimum detectable level at the receiver and the channel becomes unavailable. The CDF of the received signal magnitude is

$$\begin{aligned} F(r_0) &= P(r \leq r_0) \\ &= \int_0^{r_0} \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)} dr \\ &= \int_0^{r_0^2/(2\sigma^2)} e^{-u} du \\ &= 1 - e^{-r_0^2/(2\sigma^2)} \end{aligned} \tag{9.15}$$

Evaluating the CDF for a given value of r_0 gives the propability that the magnitude of the received signal is below r_0.

It is also convenient to perform a change of variables to transform the PDF and CDF to be functions of SNR rather than signal magnitude. The local SNR at a given time is

$$\gamma = \frac{\text{Short term average signal power}}{\text{Average noise power}} = \frac{r^2/2}{P_n} \tag{9.16}$$

where we have neglected factors related to the antenna gain and load resistance. The long term mean SNR is

$$\Gamma = \frac{\text{Long term average signal power}}{\text{Average noise power}} = \frac{\sigma^2}{P_n} \tag{9.17}$$

The PDF of the local SNR is

$$\begin{aligned} f_\gamma(\gamma) &= f_r[r(\gamma)]\frac{\partial r}{\partial \gamma} \\ &= \frac{\sqrt{2P_n\gamma}}{\sigma^2}e^{-2P_n\gamma/(2\sigma^2)}\frac{\sqrt{2P_n}}{2\sqrt{\gamma}} \\ &= \frac{P_n}{\sigma^2}e^{-\gamma P_n/\sigma^2} \\ &= \frac{1}{\Gamma}e^{-\gamma/\Gamma} \end{aligned} \tag{9.18}$$

The CDF of the SNR is is

$$P(\gamma \le \gamma_0) = \int_0^{\gamma_0} \frac{1}{\Gamma}e^{-\gamma/\Gamma}d\gamma = 1 - e^{-\gamma_0/\Gamma} \tag{9.19}$$

These distributions characterize the fluctuation of the short term or local SNR over time relative to the long term mean SNR.

An important characteristic of the Rayleigh channel model is that the PDF of the SNR given by (9.18) predicts a high probability that the field at some locations or points in time in the multipath environment will be very small in relation to the mean value of the field. The PDF of the SNR is exponential, and has its maximum value at $\gamma = 0$, or zero SNR. If the signal power or SNR in a rich multipath channel is plotted over time as the receiver or transmitter move, large drops in the signal power or SNR can be observed. This effect is referred to as Rayleigh fading. Physically, Rayleigh fading is caused by destructive of incoming waves due to phase differences between propagation paths.

9.3.2 Channel Availability

In a rich multipath environment that can be characterized by the Rayleigh channel model, because of the high likelihood of fading, or cancelation of the incoming waves from time to time at the receiver, the channel quality can be poor. If the local SNR drops below the minimum required SNR for reliable signal detection, the channel becomes unavailable to the user. The fraction of time that the total signal power is in a deep fade and falls below a given threshold is quantified by the CDF of the power or SNR.

For a Rayleigh channel, suppose that the transmit power and receiver sensitivity are such that the mean signal power σ^2 is 10 dB above the minimum detectable signal level. This means that the channel is available as long as the local average received power $r^2/2$ is greater than $0.1\sigma^2$. From Eq. (9.15) for the CDF of the power in a Rayleigh channel, the probability that $r^2/2$ is below $0.1\sigma^2$ is

$$P(r \le r_0) = 1 - e^{-r_0^2/(2\sigma^2)} = 1 - e^{0.1} \simeq .095 \tag{9.20}$$

which shows that the channel availability is 90.5%. This is a fairly low availability which may not be acceptable for many applications.

9.3.3 Ricean Channel Model

The Rayleigh channel model assumes that all propagation paths are equally likely, which occurs in a strongly scattering environment for which there is no direct path from transmitter to receiver. If there is a dominant path, typically representing line of sight propagation or a particularly strong specular reflection, then fading is less likely, as the direct path signal will generally be larger than the magnitude of the other multipath contributions and cancellation does not occur. This type of propagation environment leads to a Ricean PDF for the magnitude of the electric field at the receiver.

With a strong direct path component, the field at the receiver has the form

$$E = r_s + \sum_i Z_i \tag{9.21}$$

where r_s is the magnitude of the direct path component of the field. The dominant path is modeled as a constant and shifts E so that is is no longer a zero mean random variable.

The joint PDF of the real and imaginary parts of E is

$$f_{XY}(X,Y) = \frac{e^{-[(X-r_s)^2+Y^2]/(2\sigma^2)}}{2\pi\sigma^2} \tag{9.22}$$

where we have assumed without loss of generality that r_s is real. The joint magnitude and phase PDF is

$$\begin{aligned} f_{r\theta}(r,\theta) &= \frac{r}{2\pi\sigma^2} e^{-[(r\cos\theta - r_s)^2 + r^2\sin^2\theta]/(2\sigma^2)} \\ &= \frac{r}{2\pi\sigma^2} e^{-(r^2+r_s^2-2rr_s\cos\theta)/(2\sigma^2)} \end{aligned} \tag{9.23}$$

The magnitude distribution is

$$\begin{aligned} f_r(r) &= \int_0^{2\pi} f_{r\theta}(r,\theta)d\theta \\ &= \frac{r}{\sigma^2} e^{-(r^2+r_s^2)/(2\sigma^2)} \frac{1}{2\pi}\int_0^{2\pi} e^{rr_s\cos\theta/\sigma^2} d\theta \\ &= \frac{r}{\sigma^2} e^{-(r^2+r_s^2)/(2\sigma^2)} I_0(rr_s/\sigma^2) \end{aligned}$$

where $I_0(z) = J_0(jz)$ is a modified Bessel function of the first kind. This is a Ricean distribution. If $r_s = 0$, $I_0(0) = 1$, and the PDF reduces to a Rayleigh distribution as expected.

The strength of the direct path signal relative to the power in the other multipaths is

$$K = \frac{r_s^2}{2\sigma^2} \tag{9.24}$$

This quantity is called the Ricean K-factor. If K is small, then the channel is multipath dominated, and if K is large, the dominant path is larger than the multipath signals. We can rewrite the PDF as

$$f_r(r) = \frac{2Kr}{r_s^2} e^{-K(r^2+r_s^2)/r_s^2} I_0(2Kr/r_s) \tag{9.25}$$

The local SNR is

$$\gamma = \frac{r^2/2}{P_n} \tag{9.26}$$

The distribution function of the SNR is

$$\begin{aligned}
f_\gamma(\gamma) &= f_r[r(\gamma)]\frac{\partial r}{\partial \gamma} \\
&= \frac{\sqrt{2P_n\gamma}}{\sigma^2} e^{-K(2P_n\gamma + 2KP_n\Gamma)/(2KP_n\Gamma)} I_0(2K\sqrt{2P_n\gamma}/\sqrt{2P_nK\Gamma})\frac{\sqrt{2P_n}}{2\sqrt{\gamma}} \\
&= \frac{1}{\Gamma} e^{-(\gamma/\Gamma + K)} I_0(\sqrt{4K\gamma/\Gamma})
\end{aligned}$$

for the Ricean channel. For the Ricean channel, the channel availability increases as the dominant path becomes stronger in relation to the multipath and the K-factor becomes larger.

9.4 Temporal Channel Models

The Rayleigh and Ricean fading models provide a statistical model for the variation of the power received in a multipath environment. The propagation environment affects the received signal in other ways as well. Because of the differences in path lengths, a signal propagating along multiple paths arrives at the receiver with different time delays. We have already modeled the power cancelation through constructive and destructive interference in Eq. (9.3), but even if cancelation does not occur, the superposition of delayed copies of the transmitted waveform increases the difficulty of detecting the information carried in the modulated signal. This effect is called delay spread or time dispersion.

If there is relative motion between the transmitter, receiver, or scatterers, Doppler shifts change the frequency content of the signal, leading to frequency spreading or frequency dispersion. Because both effects are caused by relative motion of the transceivers and propagation environment, we will find that there is a close connection between the delay spread in a propagation environment and the Doppler spreading.

9.4.1 Time Delays (Delay Spreading)

One approach to modeling path delays in multipath propagation is in terms of the channel impulse response. As a function of time, the field at the receiver can be represented as

$$E(t) = \sum_{n=1}^{N} a_n(t) E^{\text{trans}}[t - \tau_n(t)] \tag{9.27}$$

where each term in the sum represents a propagation path. The argument of the transmitted electric field represents a time shift due to the propagation delay $\tau_n(t)$ for the nth reflection path from the transmitter to the receiver in the propagation environment at time t, and $a_n(t)$ is the amplitude of the path. Typically, the amplitude and delay for each path vary on a slower time scale than the modulated signal $E^{\text{trans}}(t)$.

Based on this path delay model, the channel impulse response is

$$h(t, \tau) = \sum_{n=1}^{N} a_n(t) \delta[\tau - \tau_n(t)] \tag{9.28}$$

where t is the time at which the channel is measured and τ represents signal delay. The received field can be obtained by convolving the impulse response with the transmitted field using

$$E(t) = \int h(t, \tau) E^{\text{trans}}(t - \tau)\, d\tau \tag{9.29}$$

If the transmitter, receiver, and scatterers in the propagation environment are fixed, then the path amplitudes and delays are constant. In this case, $h(t, \tau) = h(\tau)$ and the channel is time-invariant.

In a rich multipath environment, (9.28) has so many terms that the string of delta functions can be considered to be so close together as to be indistinguishable, and the impulse response can be approximated by a continuous function. Typically, we would expect longer delays to correspond to more scattering and free space propagation loss, so $h(t, \tau)$ is generally a decaying function with respect to the time delay τ. The length of the impulse response is the delay spread of the channel. In an environment with short and long propagation paths, the delay spread is large. If the paths are close in length, the delay spread is small.

9.4.2 Doppler Shifts (Frequency Spreading)

Doppler shifts can be modeled by assuming a rich multipath environment with a uniform relative velocity between the receiver and the scatterers in the environment. To simplify the analysis, we will further assume that all paths arrive in horizontal directions from uniformly distributed azimuth angles. This will lead to what is called the Clarke fast fading model.

Relative motion causes a Doppler shift for the nth path given by

$$\omega_n = kv\cos\psi_n \tag{9.30}$$

where v is the velocity of the receiver and ψ_n is the azimuth angle of arrival of the signal with respect to the direction of motion. As a function of angle of arrival, the frequency of the signal at the receiver including Doppler shift is

$$f(\psi) = f_{\max}\cos\psi + f_c \tag{9.31}$$

where f_c is the carrier frequency and

$$f_{\max} = \frac{kv}{2\pi} = \frac{v}{\lambda} \tag{9.32}$$

is the maximum Doppler shift. The radian frequency of the field arriving from the nth path is $\omega_n = 2\pi f(\psi_n)$.

Combining the fields contributed by all of the propagation paths, the field at the receiver is

$$E(t) = \sum_{n=1}^{N} E_n e^{j(\omega_n t + \phi_n)} \tag{9.33}$$

where E_n and ϕ_n are the amplitude and phase shift associated with the nth path. This is similar to (9.27) but in phasor form with an additional frequency shift $e^{j\omega_n t}$ due to the Doppler effect.

The autocorrelation function of the received field is

$$\begin{aligned}
R(\tau) &= \mathrm{E}[E(t)E^*(t+\tau)] \\
&= \mathrm{E}\left[\sum_{m=1}^{N} E_m e^{j(\omega_m t+\phi_m)} \sum_{n=1}^{N} E_n^* e^{-j[\omega_n(t+\tau)+\phi_n]}\right] \\
&= \sum_n \mathrm{E}[|E_n|^2 e^{-j\omega_n\tau}] \quad \text{(Assuming uncorrelated paths)} \\
&= \sum_n \mathrm{E}[|E_n|^2]\mathrm{E}[e^{-j\omega_n\tau}] \quad \text{(Assuming uncorrelated delay and amplitude)} \\
&= 2P_0\mathrm{E}[e^{-j\omega\tau}] \quad \text{(Assuming identically distributed paths)} \\
&= 2P_0\frac{1}{2\pi}\int_0^{2\pi} e^{-j\omega_{\max}\tau\cos\psi}\,d\psi \quad \text{(Assuming uniformly distributed arrival angles)} \\
&= 2P_0 J_0(\omega_{\max}\tau)
\end{aligned} \tag{9.34}$$

where P_0 is proportional to the received power due to the signal along one of the propagation paths.

Assuming that the transmitted signal is a single frequency tone, the power spectral density of the received signal is the Fourier transform of the autocorrelation function:

$$\begin{aligned}
S(f) &= \mathrm{F}\{R(\tau)\} \\
&= \int_{\infty}^{\infty} e^{-j2\pi f\tau} 2P_0 J_0(2\pi f_{\max}\tau)\, d\tau \\
&= 2P_0 \int_{\infty}^{\infty} e^{-j2\pi f\tau} \frac{1}{2\pi} \int_0^{2\pi} e^{-j2\pi f_{\max}\tau\cos\psi}\, d\psi\, d\tau \\
&= 2P_0 \int_0^{2\pi} d\psi\, \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j2\pi(f+f_{\max}\cos\psi)\tau}\, d\tau \\
&= 2P_0 \int_0^{2\pi} d\psi\, \delta(f + f_{\max}\cos\psi) \\
&= 2P_0 \int_0^{2\pi} d\psi\, \frac{\delta(\psi-\psi_0)}{|u'(\psi_0)|} \quad \text{where } u(\psi) = f + f_{\max}\cos\psi \text{ and } u(\psi_0) = 0 \\
&= \frac{2P_0}{|u'(\psi_0)|} \quad \text{if } |f| \le f_{\max}
\end{aligned}$$

The derivative of $u(\psi)$ at $\psi = \psi_0$ is

$$\begin{aligned}
|u'(\psi_0)| &= |-2\pi f_{\max}\sin\psi_0| \\
&= \left|-2\pi f_{\max}\sin[\cos^{-1}(-f/f_{\max})]\right| \\
&= 2\pi f_{\max}\sqrt{1-(f/f_{\max})^2}
\end{aligned}$$

This leads to the power spectrum

$$S(f) = \begin{cases} \frac{P_0}{\pi f_{\max}\sqrt{1-(f/f_{\max})^2}} & |f| \le f_{\max} \\ 0 & |f| > f_{\max} \end{cases} \tag{9.35}$$

This is the spectrum of the signal at baseband, so that the RF spectrum is shifted so that the band center is at the carrier frequency f_c. Since the transmitted signal is modulated, the actual spectrum is given by this Doppler spectrum convolved with the frequency content of the modulated signal.

As expected, the bandwidth of the spectrum is $2f_{\max}$. At the band edges $f = \pm f_{\max}$, the ideal spectrum is infinite, but in a real channel the transmitted signal has finite bandwidth, the scatterers are discrete, and the angle of arrival distribution is discontinuous, and as a result the actual observed spectrum is finite for all frequencies.

9.4.3 Coherence Time

For a given correlation function, the coherence time is the time over which the channel response is strongly correlated. If we choose a decorrelation threshold of 0.5, the coherence time can be defined by

$$\frac{R(t_{\mathrm{coh}})}{R(0)} = 0.5 \tag{9.36}$$

For the fast fading Doppler model developed above, the correlation function is given by the zeroth order Bessel function, for which $J_0(1.5) \simeq 0.5$. This leads to a coherence time estimate of

$$2\pi f_{\max} t_{\mathrm{coh}} \simeq 1.5 \quad \Rightarrow \quad t_{\mathrm{coh}} = \frac{0.24}{f_{\max}} \tag{9.37}$$

It can be seen that the larger the Doppler shift in the channel, the smaller the coherence time of the propagation environment. This is due to rapid motion of the receiver, so that only a short time is required before motion causes a large change in the propagation environment. As expected, a stationary environment has zero Doppler spreading and an infinite coherence time.

From the maximum Doppler shift (9.32), the coherence time is

$$t_{\rm coh} \simeq 0.24\frac{\lambda}{v} \tag{9.38}$$

Multiplying the coherence time by the velocity v shows that over the coherence time, the receiver has moved by a distance equal to a fraction of the wavelength λ. A rich multipath environment therefore varies in its propagation characteristics over a length scale proportional to the electromagnetic wavelength, as might be expected. A slow fading environment changes over a much larger scale and has a longer coherence time for a given rate of motion.

9.4.4 Coherence Bandwidth

The coherence bandwidth is the range of frequencies over with the channel response is strongly correlated. Over the coherence bandwidth, frequency components experience similar fading. Frequency components separated by more than the coherence bandwidth have a different response over the multipath propagation channel.

The coherence bandwidth is related to the delay spread in the propagation channel caused by differences in the lengths of the multipaths. From the channel impulse response (9.28), we can find the power corresponding to each path as a function of delay, or the power delay profile $P_h(\tau)$. The Fourier transform of the power delay profile gives the channel coherence spectrum. The channel coherence spectrum represents the correlation between the channel response at frequencies separated by a given value Δf.

The coherence bandwidth is the frequency separation Δf where the channel coherence spectrum decreases by a given threshold, such as 0.5 as was used for the coherence time estimate. This leads to the coherence bandwidth estimate

$$\mathrm{BW}_{\rm coh} \simeq \frac{1}{t_{\max}} \tag{9.39}$$

where $t_{\max}$ is the longest spread between delays in the propagation environment. The coherence bandwith is inversely related to the time delay spread of the channel. The RMS multipath delay spread can also be used to estimate the coherence bandwidth. If there is no delay spread, as is the case for a single path with no scatterers in the propagation environment, then the channel response does not change significantly over frequency and the coherence bandwidth is infinite.

9.4.5 Wide Sense Stationary Uncorrelated Scattering (WSSUS) Channels

An important class of propagation models are those for which the channel is wide sense stationary and scatterers are uncorrelated. Wide sense stationary means that while the detailed channel properties change rapidly with position and may vary widely, the statistical properties of the channel, such as coherence time and bandwidth, do not depend on time. Physically, the channel may be nonstationary in the sense that the transmitter, receiver, or scatterers are moving or changing in time, but the movement occurs in such a way that the average behavior of the channel is constant and therefore the channel is stationary in the statistical sense. Practically speaking, WSSUS means that the channel looks similar in a big picture sense from any receiver location.

Uncorrelated scattering means that the coefficients $E_n(\tau)$ and $\tau_n(\tau)$ in (9.28) are independent, so that the amplitude and delay of one path is independent of the amplitude and delay of another path. A real propagation channel will not be WSSUS over all time, but this assumption may still be valid in an approximate sense over periods of time on the order of milliseconds, seconds, or even longer depending on rates of motion. For this reason, channels that are not truly WSSUS are often modeled as stationary, as the statistical properties change on a much lower time scale or longer spatial scale than the detailed multipath structure.

9.5 Time-Angle Propagation Models

So far we have assumed that multipaths propagate towards a receiver with a uniform distribution over all arrival angles. In practical channels, multipaths may be clustered around certain strong ray paths through the environment. A more sophisticated way to measure and characterize multipath channels that models more complex distributions of ray paths is in terms of the distribution of paths by angle of arrival angle at a receiver and time delay.

We can incorporate time delays and the angle of arrival distribution by modeling the fields at the receiver according to

$$E(\overline{k},\overline{k}',t) = \sum_{n=1}^{N} a_n e^{j\phi_n}\delta(t-\tau_n)\delta(\overline{k}-\overline{k}_n)\delta(\overline{k}'-\overline{k}'_n) \tag{9.40}$$

where the nth multipath has amplitude a_n, phase ϕ_n, time of arrival τ_n, direction of arrival (DOA) $\overline{k}_n$, and direction of departure (DOD) $\overline{k}'_n$. Each term in the summation corresponds to a wave arriving at the receiver in a specific direction due to a wave radiated by the transmitter in a specific direction.

As with the channel impulse response (9.28), in a rich multipath environment the sum of delta functions in (9.40) can be approximated as a continuous function with some angular distribution of waves at the receiver. The arrival angle distribution can be interpreted as a PDF, so that large values of the distribution correspond to angles of arrival that are likely to occur, and small values correspond to angles from which few multipaths arrive. From the statistical point of view, the simplest angle of arrival distribution is uniform, so that a signal arriving from one direction is just as likely as a signal from another direction. This is known as Jakes' or Clark's channel model. A nonuniform DOA distribution might represent a channel with some moving scatterers but also fixed scatterers that reflect propagating waves in such a way that they arrive most often from a preferred range of directions. A wireless network terminal near a wall is an example of this type of channel.